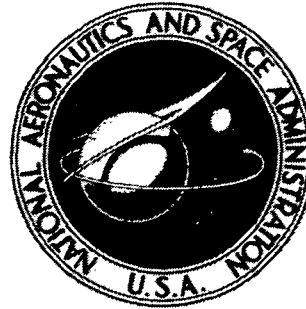


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**ESTIMATES OF TRAPPED RADIATION
ENCOUNTERED ON LOW-THRUST
TRAJECTORIES THROUGH THE
VAN ALLEN BELTS**

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Cleveland, Ohio 44135

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ESTIMATES OF TRAPPED RADIATION ENCOUNTERED ON LOW-THRUST TRAJECTORIES THROUGH THE VAN ALLEN BELTS

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SUMMARY

Estimates were made of the number of trapped protons and electrons encountered by vehicles on low-thrust trajectories through the Van Allen belts. The estimates serve as a first step in assessing whether these radiations present a problem to on-board sensitive components and payload. The integrated proton spectra $\Phi_p > E$ (E ranging from 4 to 300 MeV) and electron spectra $\Phi_e > E$ (E ranging from 0.5 to 4.25 MeV) are presented for the case of a trajectory described by a vehicle with a constant-thrust acceleration A_c equal to 10^{-3} meter per second per second. This value of acceleration corresponds to a trip time of about 54 days from low Earth orbit to synchronous orbit. It is shown that the time spent in the belts and hence the radiation encountered vary nearly inversely with the value of thrust acceleration (for values of A_c less than about 10^{-2} meter/sec²). Thus, the integrated spectral values presented for the case of $A_c = 10^{-3}$ meter per second per second can be generalized for any other value of thrust acceleration A_c (less than 10^{-2} meter/sec²) by multiplying them by the factor $10^{-3}/A_c$.

INTRODUCTION

Vehicles with low-thrust electric propulsion engines are being considered for missions from low Earth orbit to synchronous orbit or Earth escape. These spiral slowly through the Van Allen belts and encounter appreciable numbers of protons and electrons that are trapped in the belts. This radiation can be damaging to sensitive components and payload if it exceeds certain limits, in which case shielding has to be provided to reduce this radiation. Vehicle structure and on-board equipment may provide some or all of the shielding required. Reference 1 discusses the damage effects of radiations from various sources in space and from on-board nuclear power sources on various spacecraft subsystems.

As a first step in assessing this problem for any vehicle, the numbers of protons and electrons encountered during passage through the belts have to be evaluated. Vette and coworkers have constructed models of the trapped radiation environment (refs. 2 to 4) from which the proton and electron flux spectra at any position in the belts are determined. The integrated spectra of protons and electrons can then be obtained along a known trajectory. Reference 1 also describes calculations of such spectra for a fairly rapid traversal of the belts (about 1 hr) during a deep space mission.

In this report these spectra are determined for low-thrust trajectories and generalized for the case of constant thrust acceleration for all values of thrust acceleration less than about 10^{-2} meter per second per second. The integrated proton spectrum $\Phi_p > E$ has been calculated over a range of E from 4 to 300 MeV. This extensive range permits accurate evaluation of the ionization dose from the protons (either unshielded, or after passing through appreciable amounts of shielding). The integrated electron spectrum $\Phi_e > E$ is calculated for energies ranging from 0.5 to 4.25 MeV. At higher energies the electron flux drops very rapidly. The electron spectral calculations are based on a recent update of the electron environmental model (ref. 4).

RADIATION DATA

The model of the trapped proton environment used to evaluate the protons is described in references 2 and 3. These references also present orbital integration tables that give the average proton spectra encountered per day during circular orbits at various altitudes (150 to 10 000 n mi) and orbital inclinations (0° to 90°). Table 3 in reference 2, which is based on proton map AP6, gives integrated spectral data for the energy range 4 to 30 MeV, and table 3 in reference 3, based on proton map AP7, pertains to the energy range from 50 to 300 MeV.

The electrons were evaluated by using the electron environment data presented in reference 4. Table 9 in the reference gives the average electron spectra encountered per day during circular orbits at various altitudes up to 18 000 nautical miles and orbital inclinations from 0° to 90° . This table, based on electron maps AE4 (for the outer region of the belts, altitudes ≥ 10 000 n mi) and AE5 (for the inner regions), is representative of the year 1967, a period of near maximum solar activity. During such a period electron fluxes in the belts are enhanced; also the fluxes in the outer regions undergo short-term temporal variations due to magnetic storms. Reference 4 presents time-averaged fluxes (averaged over several months) as well as fluxes that have probabilities of 0.1 to 0.9 of being exceeded in the outer regions.

TRAJECTORY

For trajectories with low, constant tangential or circumferential thrust, a simplified approximate equation can be obtained (see the appendix) that presents altitude as a function of propulsion time for any constant value of thrust acceleration:

$$h = \frac{1}{\left(\frac{1}{\sqrt{R + h_0}} - \frac{A_c t}{\sqrt{\mu}} \right)^2} - R$$

where

h altitude, meters

R radius of Earth, 6.380×10^6 meters

h_0 initial orbit altitude, meters

A_c vehicle thrust acceleration, meters/sec²

t propulsion time, sec

μ gravitational parameter, 3.986×10^{11} cubic meters/sec²

For the purpose of evaluating radiation encountered in the Van Allen belts, this equation is accurate for altitudes up to synchronous orbit for values of thrust acceleration less than 10^{-2} meter per second per second.

It is apparent from the equation that the time to reach a given altitude varies inversely with the vehicle thrust acceleration. The total trip time and the radiation encountered also vary inversely with the thrust acceleration. For the calculations, a value of acceleration of 10^{-3} meter per second per second (resulting in a trip time of 54 days from a 100-n-mi orbit to synchronous orbit) was arbitrarily selected. The radiation encountered on a mission can then be generalized for any other value of acceleration A_c (A_c less than 10^{-2} meter/sec²) by multiplying the calculated values by the factor $10^{-3}/A_c$.

METHOD

During passage through most of the trapped radiation belts the vehicle circles the Earth several times a day and gains a little altitude each day. Thus, the orbit integration tables presented in references 2 to 4 give a good estimate of the radiation spectra encountered each day of the mission and were used to simplify the integration of

radiation encountered along the trajectory. For protons a set of curves was plotted from these tables, each curve showing the proton flux encountered per day having energies greater than some energy E_i as a function of altitude. Curves for values of E_i of 4, 14, 30, 50, 100, 150, 200, 250, and 300 MeV were generated. (An illustrative curve for $\phi_p > 14$ MeV is shown in fig. 1.) Then from the trajectory equation the altitude attained during each day of the mission was tabulated, and from each curve the flux encountered each day (corresponding to the daily altitudes) was determined. Table I shows such a listing for $\phi_p > 14$ MeV and includes a running summation of the protons encountered during the mission. The spectra of protons encountered on both 0° and 30° trajectory inclinations were calculated.

For electrons the spectra ($\phi_e > 0.5, 1, 2, 3, 3.5, 4$, and 4.25 MeV) encountered on 0° and 30° trajectory inclinations were similarly evaluated. For each trajectory two spectra were determined, one based on fluxes in the outer regions of the belt (altitudes $\geq 10\,000$ n mi) that are time-averaged and the other based on fluxes that have a probability of 0.1 of being exceeded. The latter is a more conservative estimate. Figure 2, showing the electron flux $\phi_e > 2$ MeV encountered per day as a function of altitude, illustrates the type of data used.

RESULTS

Proton spectra encountered on trajectories at 0° and 30° inclinations are listed in table II and plotted in figure 3. Maximum numbers of protons are encountered at 0° inclination. Slightly more than twice as many protons are encountered at 0° than at 30° inclinations.

Above an altitude of 7000 nautical miles the fluxes of protons having energies greater than 4 MeV decrease sufficiently so that they contribute only slightly to the total amount of protons encountered on missions beyond this altitude.

Included in table II are the ranges in aluminum corresponding to each proton energy listed. (The range at any energy E_i is that amount of material that will stop all protons having energies less than E_i .)

Electron spectra for trajectories at 0° and 30° inclinations are listed in table III and plotted in figure 4. Electrons encountered at 0° inclination are about 60 percent greater than those at 30° inclination. Also, more conservative estimates of short-term temporal variations in the outer regions of the belts (fluxes having probabilities equal to 0.1 of being exceeded) result in about 40 percent more electrons encountered than estimates from time-averaged fluxes.

From figure 2 it can be seen that, although the electron flux begins to fall off with altitude, fairly high values of electron flux extend out to synchronous orbit altitude. Extrapolation of data beyond this orbit indicates that the flux decrease with altitude is

sufficient so that only a small fraction of additional electrons is encountered on missions beyond synchronous orbit. At synchronous orbital altitude, however, the fluxes can contribute a significant integrated flux if the period spent at this altitude is large. Table IV lists the electron fluxes encountered daily in synchronous orbit. These values, of course, are not dependent on vehicle thrust acceleration.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, February 14, 1973,
503-05.

APPENDIX - DERIVATION OF APPROXIMATE TRAJECTORY EQUATION*

Consider the case where

- (1) The vehicle is initially in a circular orbit about the Earth.
- (2) The thrust and mass of the vehicle (and hence the thrust acceleration) are constant.
- (3) The thrust acceleration is small compared to the local acceleration due to gravity, so that the orbit remains approximately circular.
- (4) The thrust is directed circumferentially (it will also be nearly tangential because the orbit is almost circular).

For these conditions

$$\text{torque} = Fr = \frac{d(mvr)}{dt} \quad (1)$$

where

F thrust

r distance from center of Earth

m mass

v velocity

t time thrust has been applied

Because the orbit is nearly circular, the centrifugal force is nearly equal to the gravitational force,

$$\frac{mv^2}{r} \approx \frac{m\mu}{r^2}$$

where μ is the gravitational parameter, and

$$v \approx \sqrt{\frac{\mu}{r}} \quad (2)$$

From equation (1)

$$\frac{F}{m} \equiv A_c = \frac{1}{r} \frac{d(vr)}{dt}$$

* Provided by Frank J. Hrach, NASA Lewis Research Center.

Substituting for v from equation (2) gives

$$A_c = \frac{1}{r} \frac{d(\sqrt{\mu} r)}{dt} = \frac{\sqrt{\mu}}{2r^{3/2}} \frac{dr}{dt}$$

from which

$$\int_0^t dt = \frac{\sqrt{\mu}}{2A_c} \int_{r_0}^r \frac{dr}{r^{3/2}}$$

$$t = \frac{\sqrt{\mu}}{A_c} \left(\frac{1}{\sqrt{r_0}} - \frac{1}{\sqrt{r}} \right)$$

or

$$r = \frac{1}{\left(\frac{1}{\sqrt{r_0}} - \frac{A_c t}{\sqrt{\mu}} \right)^2} \quad (3)$$

Now

$$\left. \begin{aligned} r_0 &= R + h_0 \\ r &= R + h \end{aligned} \right\} \quad (4)$$

where

R radius of Earth

h_0 initial orbit altitude

h orbit altitude

Using equations (4) in equation (3) yields

$$h = \frac{1}{\left(\frac{1}{\sqrt{R + h_0}} - \frac{A_c t}{\sqrt{\mu}} \right)^2} - R \quad (5)$$

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TABLE I. - CALCULATION OF INTEGRATED PROTON FLUX $\phi_p > 14$ MeV DURING MISSION

[Orbit inclination, 0° ; thrust acceleration, 10^{-3} meter/sec².]

Time, t, day	Altitude, h, n mi	Proton flux, $\phi_p > 14$ MeV, $\frac{\text{protons}}{(\text{cm}^2)(\text{day})}$	Summation of proton flux, $\sum (\phi_p > 14 \text{ MeV}) =$ $\phi_p > 14 \text{ MeV},$ protons/cm ²	Time, t, day	Altitude, h, n mi	Proton flux, $\phi_p > 14 \text{ MeV},$ $\frac{\text{protons}}{(\text{cm}^2)(\text{day})}$	Summation of proton flux, $\sum (\phi_p > 14 \text{ MeV}) =$ $\phi_p > 14 \text{ MeV},$ protons/cm ²
1	180	-----	-----	20	2408	10.7×10^9	73.4×10^9
2	263	-----	-----	21	2579	10.0	83.4
3	348	-----	-----	22	2757	8.8	92.2
4	437	1.1×10^7	1.1×10^7	23	2943	7.7	99.9
5	528	6.0	7.1×10^7	24	3137	6.7	106.6
6	623	1.7×10^8	1.8×10^8	25	3341	5.9	112.5
7	722	3.8	5.6	26	3554	5.3	117.8
8	824	7.6	13.2	27	3777	4.7	122.5
9	929	1.2×10^9	2.52×10^9	28	4011	4.1	126.6
10	1039	1.8	4.32	29	4257	3.0	129.6
11	1153	2.55	6.87	30	4515	1.7	131.3
12	1272	3.4	10.3	31	4786	9.5×10^8	141.3
13	1395	4.2	14.5	32	5071	4.7	141.8
14	1522	5.2	19.7	33	5372	2.4	142.0
15	1656	6.2	25.9	34	5689	1.3	142.1
16	1794	7.3	33.2	35	6023	6.6×10^7	142.2
17	1938	8.8	42.0	36	6375	3.2	142.2
18	2088	10.0	52.0	37	6748	1.5	142.2
19	2245	10.7	62.7	38	7142	$< 10^7$	142.2

TABLE II. - PROTON SPECTRA ENCOUNTERED IN
VAN ALLEN BELTS ON LOW-THRUST
TRAJECTORIES

[Vehicle thrust acceleration $A_c = 10^{-3}$ meter/sec²; for
other values of A_c , $\Phi_p > E_i = (\text{table value}) 10^{-3}/A_c$.]

Orbit inclination, deg	Energy, E_i , MeV	Proton spectra, $\Phi_p > E_i$, protons/cm ²	Range (E_i), ^a g/cm ² of aluminum
0	4	2.9×10^{12}	0.035
	14	1.4×10^{11}	.31
	30	2.7×10^{10}	1.2
	50	1.3	2.9
	100	7.6×10^9	9.9
	150	4.7	20.2
	200	2.9	33.1
	250	1.8	48.3
	300	1.1	65.3
30	4	1.3×10^{12}	0.035
	14	6.0×10^{10}	.31
	30	1.1	1.2
	50	6.2×10^9	2.9
	100	3.7	9.9
	150	2.2	20.2
	200	1.3	33.1
	250	8.2×10^8	48.3
	300	5.0	65.3

^aRange (E_i) is amount of material that will completely stop
all protons having energies less than E_i .

TABLE III. - ELECTRON SPECTRA ENCOUNTERED IN VAN ALLEN BELTS

ON LOW-THRUST TRAJECTORIES

[Vehicle thrust acceleration $A_c = 10^{-3}$ meter/sec²; for other values of A_c ,
 $\Phi_e > E_i \approx (\text{table value}) 10^{-3}/A_c$.]

Orbit inclination, deg	Energy, E_i , MeV	Electron spectra, $\Phi_e > E_i$, electrons/cm ²		Range (E_i), ^a g/cm ² of aluminum
		Based on time-averaged fluxes	Based on fluxes having probability of 0.1 of being exceeded	
0	0.5	2.8×10^{13}	3.8×10^{13}	0.22
	1.0	8.1×10^{12}	1.1	.55
	2.0	1.7	2.1×10^{12}	1.2
	3.0	1.8×10^{11}	2.3×10^{11}	1.9
	3.5	3.1×10^{10}	4.8×10^{10}	2.2
	4.0	3.4×10^9	5.1×10^9	2.5
	4.25	5.9×10^8	8.6×10^8	2.6
30	0.5	1.6×10^{13}	2.2×10^{13}	0.22
	1.0	4.8×10^{12}	6.9×10^{12}	.55
	2.0	9.3×10^{11}	1.2	1.2
	3.0	1.1	1.4×10^{11}	1.9
	3.5	2.2×10^{10}	3.1×10^{10}	2.2
	4.0	2.4×10^9	3.2×10^9	2.5
	4.25	4.4×10^8	5.5×10^8	2.6

^aRange (E_i) is amount of material that will completely stop all electrons having energies less than E_i .

TABLE IV. - ELECTRON FLUXES ENCOUNTERED DAILY
WHILE ORBITING AT SYNCHRONOUS ALTITUDE

Orbit inclination, deg	Energy, E_i , MeV	Electron flux, $\varphi_e > E_i$, electrons/(cm ²)(day)	
		Time-averaged flux	Flux with probability of 0.1 of being exceeded
0	0.5	2×10^{11}	5×10^{11}
	1.0	5×10^{10}	10^{11}
	2.0	3×10^9	6×10^9
	3.0	10^8	2×10^8
	3.5	10^7	2×10^7
	4.0	$< 10^5$	$< 10^5$
	4.25	~ 0	~ 0
30	0.5	9×10^{10}	2×10^{11}
	1.0	2×10^{10}	4×10^{10}
	2.0	9×10^8	2×10^9
	3.0	3×10^7	7×10^8
	3.5	3×10^6	8×10^6
	4.0	$< 10^5$	$< 10^5$
	4.25	~ 0	~ 0

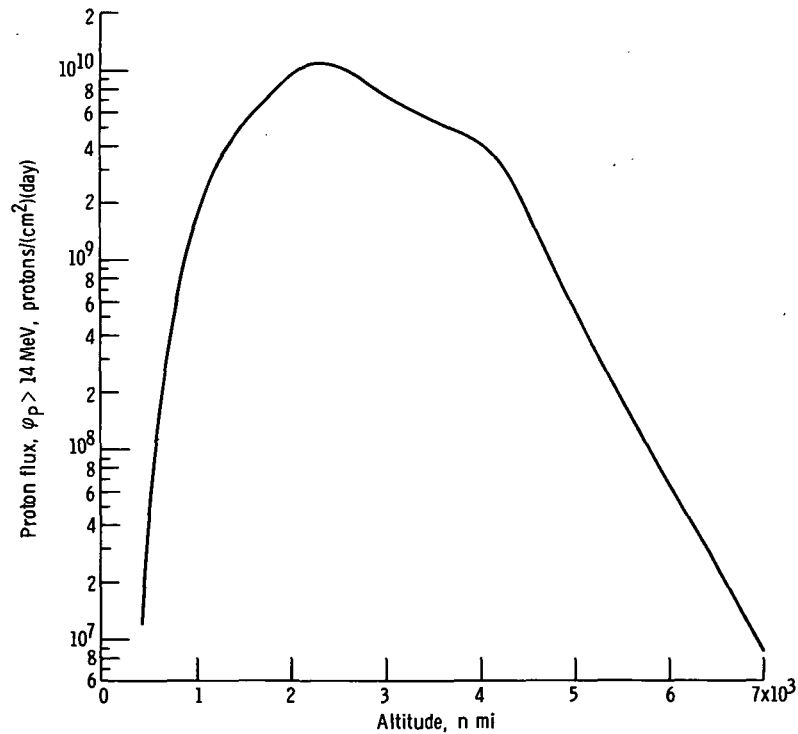


Figure 1. - Proton flux $\varphi_p > 14$ MeV encountered per day in circular orbit at various altitudes. Orbit inclination, 0° . (From ref. 2, table 3.)

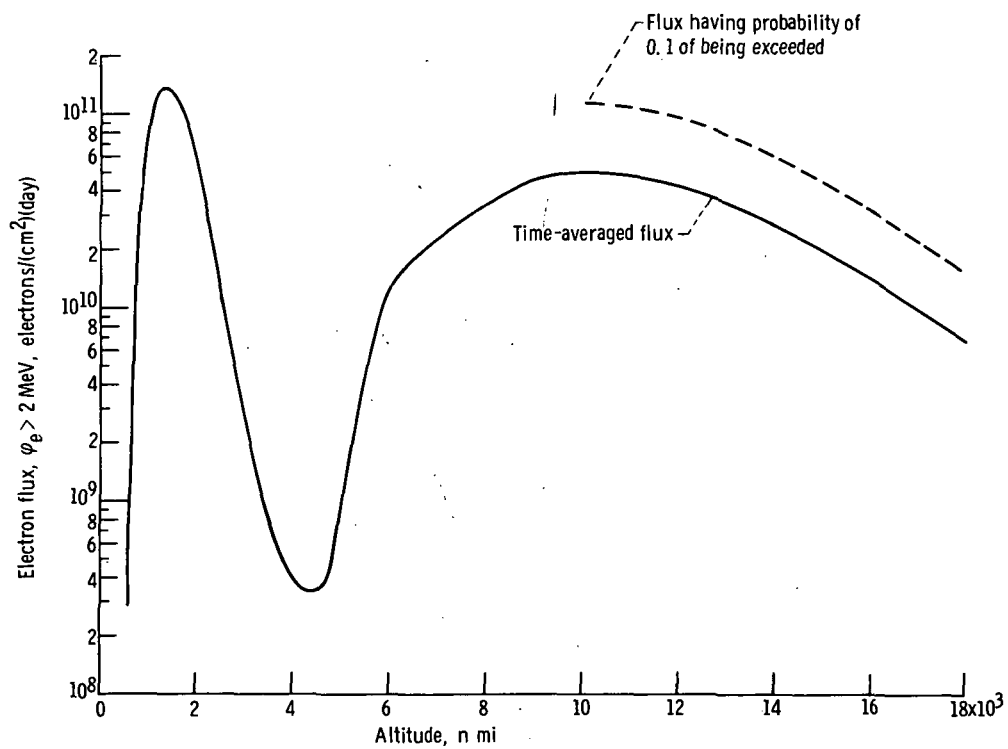


Figure 2. - Electron flux $\phi_e > 2$ MeV encountered per day in circular orbit at various altitudes. Orbit inclination, 0° . (From ref. 4, table 9.)

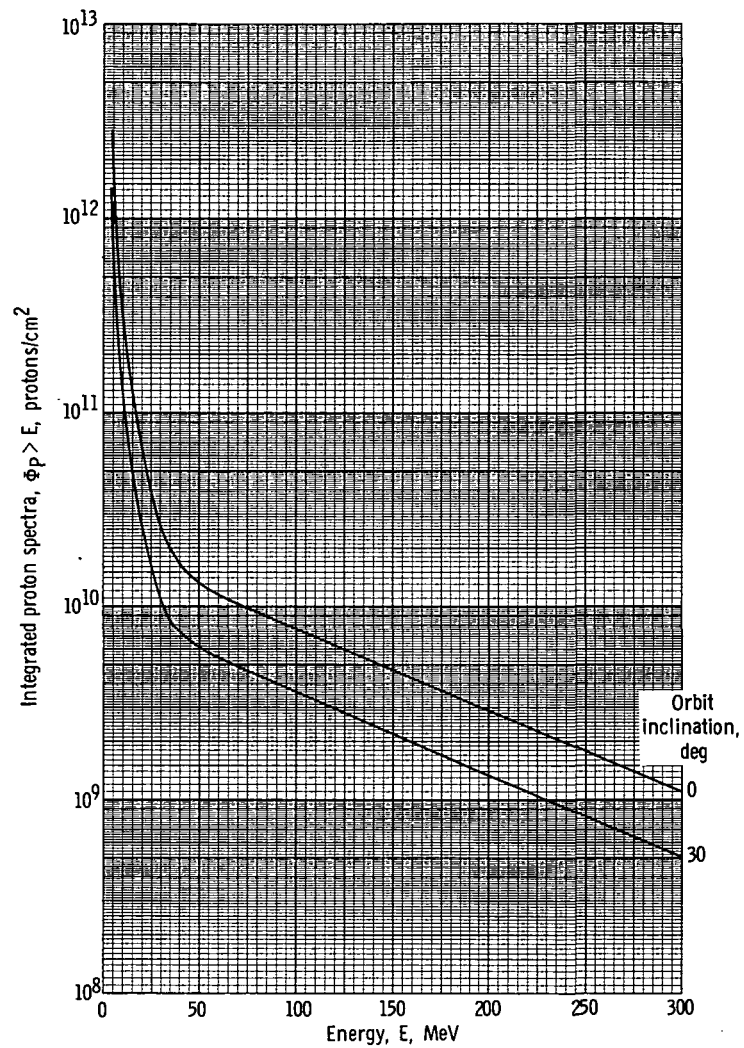


Figure 3. - Integrated proton spectra encountered on trajectories from low Earth orbit to synchronous orbit. Thrust acceleration, 10^{-3} meter/sec².

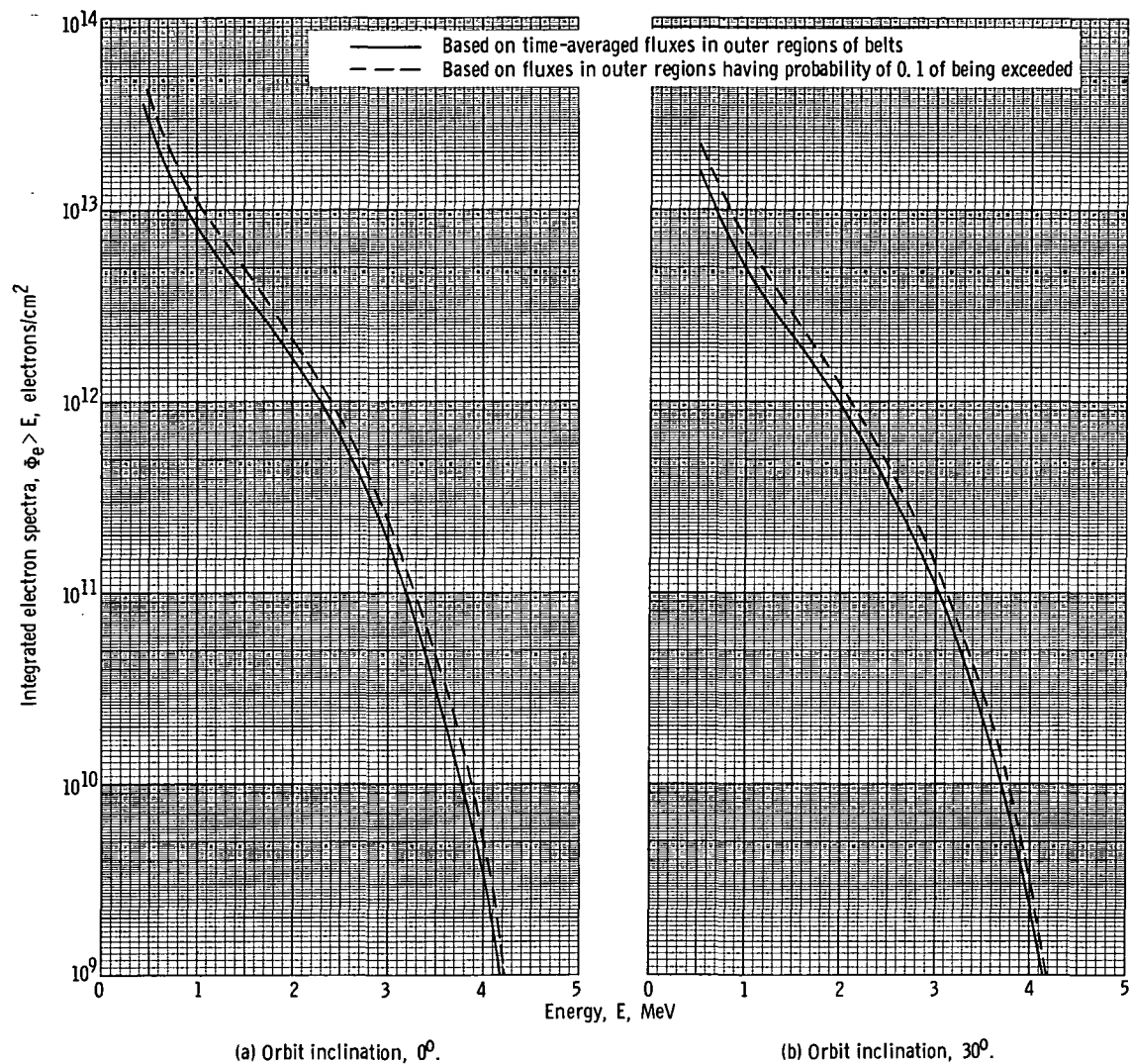


Figure 4. - Integrated electron spectra encountered on trajectories from low Earth orbit to synchronous orbit. Thrust acceleration, 10^{-3} meter/sec².

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